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# THE VALUATION OF AIRPORT SLOTS

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## I. INTRODUCTION

**I**N ORDER TO SERVE coordinated airports, an airline must **l**own take-off and landing rights (slots).<sup>1</sup> These rights allow an airline to use the airport's runway capacities at a specific time on a specific day.<sup>2</sup> Take-off and landing rights are allocated by national authorities free of charge, but the demand for slots far exceeds the supply.<sup>3</sup> Under certain circumstances, the airlines therefore have a strong incentive to acquire slots by purchasing them from other airlines. Such slot trading is allowed in the United States and the United Kingdom.<sup>4</sup> In the other member

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<sup>1</sup> INT'L AIR TRANSP. ASS'N, WORLDWIDE SCHEDULING GUIDELINES 4 (12th ed. 2005).

<sup>2</sup> See *id.* at 11.

<sup>3</sup> TRANSP. RESEARCH CTR., TRANSP. SERVS.: THE LIMITS OF (DE)REGULATION 22 (2004); David Starkie, *Allocating Airport Slots: a Role for the Market?*, 4 J. AIR TRANSP. MGMT. 111, 111 (1998).

<sup>4</sup> Starkie, *supra* note 3, at 112–13.

states of the European Union slot trading is not officially legalized yet, but is, on the other hand, not prosecuted anymore.<sup>5</sup>

If an airline wishes to purchase a slot, it must, first of all, determine its value. In this context, the aim is not to calculate an objective value at market equilibrium, as in the case of valuation models in financial theory,<sup>6</sup> but to determine the subjective value in use (VIU) of the slot for the airline in question. Knowing the VIU of a slot is of considerable importance to an airline because it enables it to establish the maximum price it could pay for the slot without the transaction being disadvantageous.<sup>7</sup> The greater extent to which the price of the slot is below its VIU, the more advantageous the purchase of the take-off or landing right for the airline. If, on the other hand, the price of the slot exceeds its VIU, the transaction is disadvantageous and should not

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<sup>5</sup> See Starkie, *supra* note 3, at 114. For a detailed analysis of slot trading in the United States, see generally Sabine J. Langner, *Contractual Aspects of Transacting in Slots in the United States*, 2 J. AIR TRANSP. MGMT. 151 (1995); Starkie, *supra* note 3; David Starkie, *Developments in Transport Policy—The US Market in Airport Slots*, 28 J. TRANSP. ECON. & POL'Y 325, 325 (1994). On slot trading in the United Kingdom and a discussion of the buying and selling of slots in the European Union, see generally Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions on the Application of Regulation (EEC) No 95/93 on Common Rules for the Allocation of Slots at Community Airports, as amended COM (2008) 227 final (Apr. 30, 2008); Matthias Kilian, *The Development of the Regulatory Regime of Slot Allocation in the EU*, (2004) in AIRPORT SLOTS: INTERNATIONAL EXPERIENCES AND OPTIONS FOR REFORM 252 (Achim I. Czerny et al. Eds., 2008) [hereinafter *Slot Allocation*]; CIVIL AVIATION AUTHORITY, THE IMPLEMENTATION OF SECONDARY SLOT TRADING (2001); CIVIL AVIATION AUTHORITY, INTRODUCING COMMERCIAL ALLOCATION MECHANISMS: THE UK CIVIL AVIATION AUTHORITY'S RESPONSE TO THE EUROPEAN COMMISSION'S STAFF WORKING PAPER ON SLOT REFORM (2004); Ruwantissa I.R. Abeyratne, *Management of Airport Congestion Through Slot Allocation*, 6 J. AIR TRANSP. MGMT. 29 (2000); Tom C. Bass, *Infrastructure Constraints and the EC*, 1 J. AIR TRANSP. MGMT. 145 (1994); Matthias Kilian, *Der Handel mit Slots—Die Entscheidung des High Court im Verfahren Regina vs. Airport Co-Ordination Ltd.*, 23 TRANSPR 159 (2000) (F.R.G.).

<sup>6</sup> See, e.g., TOM COPELAND, TIM KOLLER & JACK MURRIN, VALUATION: MEASURING AND MANAGING THE VALUE OF COMPANIES (3d ed. 2000); John Stonier, *Airline Long-Term Planning Under Uncertainty: The Benefits of Asset Flexibility Created Through Product Commonality and Manufacturer Lead Time Reductions*, in REAL OPTIONS AND BUSINESS STRATEGY 135 (Lenos Trigeorgis ed., 1999); Fischer Black & Merton Scholes, *The Pricing of Options and Corporate Liabilities*, 81 J. POL. ECON. 637 (1973); John C. Cox, Stephen A. Ross, & Mark Rubinstein, *Option Pricing: A Simplified Approach*, 7 J. FIN. ECON. 229 (1979).

<sup>7</sup> See Thomas Hering, Michael Olbrich & Martin Steinrücke, *Valuation of Start-up Internet Companies*, 33 INT'L J. TECH. MGMT. 406–19 (2006); Manfred Jürgen Matschke, *Der Gesamtwert der Unternehmung als Entscheidungswert*, 24 BETRIEBSWIRTSCHAFTLICHE FORSCHUNG UND PRAXIS 146–61 (1972).

be conducted. In light of the highly individualized market for slot trading which is characterized by extensive excess demand—i.e. the acquiring company is not just a price-taker—it is particularly important to identify the subjective price limit because, in some cases, airlines have to spend up to several million euros or U.S. dollars in order to buy a slot.<sup>8</sup> In other words, it is a substantial investment, especially for small and medium-sized companies. Consequently, maximum price determination does not remain only an academic concern from the perspective of valuation theory, but is also of practical interest to the respective company in the mostly bilateral negotiation process.

The importance of appropriate slot valuation, especially to an airline's capital budgeters, becomes apparent when taking a look at recent balance sheets of firms in the airline industry. In 2007, for example, Continental Airlines acquired slots at London's Heathrow Airport for a total amount of \$116 million.<sup>9</sup> Both in 2007 and 2008, the amounts invested in international slots represented 26% of the company's total capital expenditure, with the remainder consisting of the acquisition cost of new aircraft and related support equipment.<sup>10</sup> Identifying the impact of slot valuation on periodical cash flows from the perspective of an airline as large as Continental Airlines indicates that investment in take-off and landing slots is of even greater relative importance to smaller airlines that intend to serve such airports.

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<sup>8</sup> Current prices paid for slots at important international airports amount to double-digit million euro/dollar sums. For example, it is reported that Alitalia sold several slot bundles at London's Heathrow Airport in 2007 for a total of 92 million euros to competitors such as British Airways, Continental Airlines, and US Airways. Kevin Done, *Alitalia Sells Heathrow Slots*, FIN. TIMES, Dec. 26, 2007, <http://www.ft.com/cms/s/0/0aea331e-b3a0-11dc-a6df-0000779fd2ac.html>. It is furthermore reported that, among this total amount, over 30 million euros were paid for one pair of take-off and landing rights. *Id.* Similarly, Continental Airlines acquired two pairs of take-off and landing rights for \$116 million (\$58 million per bundle) at Heathrow Airport in 2007. Kevin Done, *Continental Pays Heathrow Record*, FIN. TIMES, Mar. 3, 2008, <http://www.ft.com/cms/s/0/b6a47274-e955-11dc-8365-0000779fd2ac.html> [hereinafter *Continental Pays Heathrow*]. For an analysis of the prices paid for slots, see Severin Borenstein, *The Evolution of U.S. Airline Competition*, 6 J. ECON. PERSPECTIVES 45 (1992); *Slot Allocation*, *supra* note 5.

<sup>9</sup> *Continental Pays Heathrow*, *supra* note 8.

<sup>10</sup> For further information, see Cont'l Airlines, Inc. Annual Report (Form 10-K), at 49, 79 (Feb. 11, 2008); Cont'l Airlines, Inc. Annual Report (Form 10-K), at 55, 91 (Feb. 13, 2009). Similar amounts can be found in annual reports of other airlines. See, e.g., BRITISH AIRWAYS PLC., 2007/08 ANNUAL REPORT & ACCOUNTS 16, 101 (2008). See also DEUTSCHE LUFTHANSA AG, ANNUAL REPORT 2008 155 (2009).

Although slot trading is widely discussed in literature, there is, to date, no analysis of how to determine the subjective VIU of a take-off or landing right. The present article aims to close this gap by examining how an airline should proceed in order to measure a slot's value. For this purpose, two valuation models are presented in Section II, which are specified by examples of calculations for specific valuation problems that occur in practice in Section III.

## II. VALUATION MODELS

### A. PRESENT VALUE TECHNIQUE

In Section I, reference was made constantly to the value of "a" slot. However, a closer look at the situation reveals that one single slot is generally useless to an airline because, in addition to a take-off right at Airport A, it needs a landing right at Airport B. Also, to enable the aircraft to fly back home, a take-off right at Airport B and a landing right at Airport A are needed (or even more slots, if the airline does not fly back directly from B to A but, for example, from B via C back to A). That is, in order to value slots, even for the most simple case, it is generally necessary to consider a bundle of at least four slots (Airport A: take-off and landing, Airport B: landing and take-off), and to measure the VIU, not of one single slot, but of this bundle of slots.<sup>11</sup> In this context, the earnings (net cash flows)  $CF_t$  that the airline will gain through the bundle of slots in future periods  $t$  are discounted to the present day.<sup>12</sup> These net cash flows derive from the periodical turnover  $T_t$  earned from passenger and freight transport, the expenses associated with the use of the aircraft  $EP_t$  (for example, fuel and personnel costs) and airport-related expenses  $EA_t$  (such as airport charges and gate leases):

$$(1) \quad CF_t = T_t - EP_t - EA_t.$$

Characteristic of the valuation of slots is the fact that the lifetime of this asset is generally unlimited. As long as the airline uses the take-off and landing capacity of a slot at least to an extent of 80% during a flight period, the take-off and landing right does not expire ("use it or lose it rule"). Therefore, in the

<sup>11</sup> See Hans-Jürgen Ewers et al., *Möglichkeiten der besseren Nutzung von Zeitnischen auf Flughäfen (Slots)*, in DEUTSCHLAND UND DER EU 15 (2001) (unpublished working paper, on file with Technische Universität Berlin).

<sup>12</sup> See GÜNTER JAENSCH, WERT UND PREIS DER GANZEN UNTERNEHMUNG 23–33 (1966).

event that the airline does not plan to sell the slot in the future, the valuation time horizon is indefinite and hence differs from those of other capital expenditures. However, if the airline does not plan to use the take-off and landing rights itself, but to lease them to another company for certain periods (which is possible in the United States), the periodic net cash flows are the earnings gained from the leasing contract  $L_t$ :

$$(2) \text{ CF}_t = L_t.$$

If the airline aims to sell the bundle of slots at the end of its periods of usage to another company, the surplus of the last period  $t = n$  is obtained from the earnings according to Equation (1) plus the net turnover  $NT_n$  that will be realized by selling the slots at the end of period  $n$ :

$$(3) \text{ CF}_n = T_n - \text{EP}_n - \text{EA}_n + \text{NT}_n.$$

With discount rate  $i$ , periods of usage  $t \in \{1, \dots, n\}$  and exclusive use by the airline itself (that is, no lease), the VIU of the slot bundle is calculated by:

$$(4) \text{ VIU} = \sum_{t=1}^n \frac{T_t - \text{EP}_t - \text{EA}_t}{(1+i)^t} + \frac{\text{NT}_n}{(1+i)^n}.$$

With the use of equations (1) and (3), this can be formulated more simply as:

$$(5) \text{ VIU} = \sum_{t=1}^n \frac{\text{CF}_t}{(1+i)^t}.$$

The discount rate  $i$  that is used in this context for calculating the present value is the internal rate of return (IRR) of the best alternative investment that the airline could make with the money needed to purchase the slot bundle, for example the purchase of another aircraft or the redemption of a credit.<sup>13</sup> Consequently, not only the forecast of future cash flows generated by the slot bundle, but also the determination of the relevant discount rate has to be carried out from a strict subjective perspective. This constitutes the central characteristic of the models presented in this article. Unlike this approach, valuation models in financial theory are typically based on the market equilibrium assumption in order to determine investment alternatives, capital structure, risk disposition, and, hence, the discount rate. When applying these models, the actual investment and financing alternatives of the specific airline are not taken

<sup>13</sup> See THOMAS HERING, INVESTITIONSTHEORIE (2d ed. 2003).

into account. In consequence, values that are calculated based on financial theory typically do not represent subjective maximum prices.

When the VIU of the slot bundle has been determined in the manner shown above, the airline gains information about the maximum price it can pay for the four slots together, without the transaction being disadvantageous. That is, the sum of the prices  $P_s$  that the airline pays for the four slots ( $S \in \{1, 2, 3, 4\}$ ) must not exceed the VIU of the slot bundle:<sup>14</sup>

$$(6) \sum_{s=1}^4 P_s \leq \text{VIU}.$$

The calculation of the maximum price can be demonstrated by means of the following simple example:

An airline considers the acquisition of a take-off and landing right at Airport A and a take-off and landing right at Airport B. The company forecasts that flying between A and B will lead to periodic net cash flows of three million euros that remain constant over the future periods one to three. For the sake of simplicity, we assume just three future periods of slot usage. The company knows that the return on its best alternative investment is 4%. The VIU of the slot bundle at Airports A and B is then calculated as:

$$\begin{aligned} \text{VIU} &= \sum_{t=1}^3 \frac{3 \text{ million euros}}{(1.04)^t} \\ &= 8.33 \text{ million euros.} \end{aligned}$$

The sum of the prices  $P_s$  of the slot bundle at Airports A and B must not exceed 8.33 million euros.

If the airline knows the IRR of the best alternative investment, the present value method is preferred to linear programming (presented in Section II. B.) because no further assumptions relating to the application of linear programming need to be specified. Thus, valuation complexity is reduced if the present value

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<sup>14</sup> Once the VIU is determined using the present value technique, it can also easily be applied to the decision to sell a bundle of slots. For this purpose, the VIU characterizes the minimum price for which the airline should sell the slot bundle in order to prevent a disadvantageous situation. Therefore, the following condition holds:

$$\sum_{s=1}^4 P_s \geq \text{VIU}.$$

method can be applied. However, in reality, an airline frequently does not know the best alternative investment compared to the purchase of slots because, due to its size or group structure, it has a complex program of financing and investment options. If the airline lacks information on the best alternative investment, it also does not know the IRR of this investment. This is when linear programming can be applied.

## B. LINEAR PROGRAMMING

In a situation in which the present value technique cannot be used because the airline does not know the IRR of the best alternative investment, valuation must be based on a model that is not dependent on this information. The ZGPM (Zustands-Grenzpreismodell, state marginal price model) is just such a model.<sup>15</sup> It determines the VIU of a bundle of slots with the aid of linear programming.<sup>16</sup> Linear programming enables the airline to determine the maximum price of the slot bundle based on the company's actual investment and financing alternatives without the need to discount future cash flows. Rather, the model computes the maximum amount that can be invested with the airline reaching the identical wealth level that is achieved if the acquisition is not conducted.

For the airline, the following assumptions are made. Its planning period extends over  $n$  possible future states. In the baseline situation (present state  $t = 0$ ), there are  $m$  investment or financing objects which the airline may choose individually or combine. Each object  $j$  is characterized by the state-contingent cash stream  $g_j = (g_{j0}, g_{j1}, \dots, g_{jt}, \dots, g_{jn})$  with  $g_{jt}$  being the cash flow of object  $j$  in state  $t$ . The decision variable  $x_j$  indicates how often the object (investment/financing alternative)  $j$  is undertaken. For the variables  $x_j$ , there may be upper bounds  $x_j^{\max}$  (which can also be  $\infty$ ). Moreover, for every state  $t$ , there is an autonomous cash flow  $b_t$ , that can be negative, zero, or positive,

<sup>15</sup> See Thomas Hering, *Das allgemeine Zustands-Grenzpreismodell zur Bewertung von Unternehmen und anderen unsicheren Zahlungsströmen*, 60 DIE BETRIEBSWIRTSCHAFT 362 (2000).

<sup>16</sup> See *id.*; see also JAENSCH, *supra* note 12, at 664–65; Hans M. Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems* (1963); Herbert Hax, *Investitions—und Finanzplanung mit Hilfe der Linearen Programmierung*, 16 SCHMALENBACHS ZEITSCHRIFT FÜR BETRIEBSWIRTSCHAFTLICHE FORSCHUNG 430 (1964); Helmut Laux & Günter Franke, *Zum Problem der Bewertung von Unternehmungen und anderen Investitionsgütern*, 13 UNTERNEHMENSFORSCHUNG 205 (1969); Matschke, *supra* note 7.



and that is in no way dependent on the objects to be assessed.<sup>17</sup> If the airline pursues the goal of maximizing its wealth, it will strive for the greatest possible sum asset value (AV) of weighted withdrawals, where the cash flow  $G_t$  in state  $t$  enters into the objective function with weight  $w_t$ . The  $n+1$  liquidity constraints ensure that in each state  $t$ , the sum of all cash flows remains positive or zero. The variables  $G_t$  and  $x_j$  are also limited to positive values or zero. All in all, the "baseline program" (the combination of financing and investment options that maximizes the airline's success without the purchase of the slot bundle in question) is derived from the following linear optimization approach:

$$\begin{aligned} \max. \text{ AV; } \text{AV} &= \sum_{t=0}^n w_t \cdot G_t \\ - \sum_{j=1}^m g_{jt} \cdot x_j + G_t &\leq b_t \quad \forall t \in \{0, 1, \dots, n\} \\ x_j &\leq x_j^{\max} \quad \forall j \in \{1, 2, \dots, m\} \\ G_t, x_j &\geq 0 \quad \forall t \text{ and } j \end{aligned}$$

The simplex algorithm allows an easy calculation of the baseline program resulting in an optimal AV\*. Then, buying the bundle of slots at a price  $p$  is only economically viable if it at least yields the optimal AV\* of the baseline program.

If the airline buys the bundle of slots, it thus receives its state-contingent cash stream  $g_k = (0, g_{k1}, g_{k2}, \dots, g_{kt}, \dots, g_{kn})$  and pays (in  $t = 0$ ) the price  $p$ . The VIU must then be found, that is, the maximum price  $p$  that the airline can just afford to pay, without the slot acquisition putting the company in a worse position than if, instead, it had implemented the available baseline program. In this manner,  $p$  must consequently be maximized, taking into account the restrictions of the original decision environment, extended by the cash stream from using the acquired bundle of slots and subject to the additional condition of not violating AV\*. The "valuation program" (the combination of financing and investment options that includes the acquired slot bundle in question) is calculated by the following linear optimization approach:

<sup>17</sup> The cash flow  $b_t$  is thus obtained from established and regular payments such as salaries, ongoing revenues, or debt servicing.

max. VIU; VIU :=  $p$

$$-\sum_{j=1}^m g_{j0} \cdot x_j + G_0 + p \leq b_0$$

$$-\sum_{j=1}^m g_{jt} \cdot x_j + G_t \leq b_t + g_{kt} \quad \forall t \in \{1, 2, \dots, n\}$$

$$-\sum_{t=0}^n w_t \cdot G_t \leq -AV^*$$

$$x_j \leq x_j^{\max} \quad \forall j \in \{1, 2, \dots, m\}$$

$$G_t, p, x_j \geq 0 \quad \forall t \text{ and } j$$

Again, the simplex algorithm generates the optimal solution and thus provides not only the maximum price  $p^*$  (that is, the VIU), but also the airline's optimal investment and financing program, restructured through including earnings from the slot bundle in exchange for the acquisition price  $p^*$ .

The manner in which the VIU of a bundle of slots can be calculated by the ZGPM is demonstrated below by means of an example, in which we consider four future states that are interpreted as annual points in time. In order to reduce the complexity of the example we assume, in addition to this short planning horizon of four years, that the airline has only a few investment and finance options. First, the airline can invest in a tangible asset  $T$  (for example, an acquisition of another aircraft) which is associated with the payment stream  $(-80, 40, 40, 40)$  (in million euros). Second, the airline is able to invest money in financial assets  $F$  that promise a return of 5% per annum. Furthermore, the airline can borrow money  $M$  at an interest rate of 10% per annum. The airline's business generates annual autonomous cash flows  $b_t$  amounting to thirty million euros. The objective is to compute the VIU, which is the maximum price  $p^*$  the airline could pay for the bundle of four slots. Table 1 gives an overview of the data relating to this example.

**Table 1. Data used in the example**

Year $t$	T (Tangible asset in million euros)	$M_0$ (Finance cash flows)	$M_1$ (")	$M_2$ (")	$F_0$ (Financial asset)	$F_1$ (")	$F_2$ (")	$b_i$ (Autonomous cash flow in million euros)
0	-80	1			-1			30
1	40	-1.1	1		1.05	-1		30
2	40		-1.1	1		1.05	-1	30
3	40			-1.1			1.05	30
Limit	1	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	

In the baseline program, the airline proceeds with the tangible asset investment (acquisition of another aircraft), which is, in the first year, financed partly by means of a loan. The surpluses gained in the succeeding years are—after repaying the loan—invested in financial assets. At the end of the third year, the airline achieves an asset value of 160.0375 million euros. In Table 2, the described baseline program is shown with all cash streams that occur during the four years.

**Table 2. Baseline program**

Year	$t = 0$	$t = 1$	$t = 2$	$t = 3$
Tangible asset T	-80	40	40	40
Autonomous cash flow $b_i$	30	30	30	30
Loan L	50	-55		
Financial asset $F_1$		-15	15.75	
Financial asset $F_2$			-85.75	90.0375
Asset value				<b>160.0375</b>

With regard to the valuation program, this means that the airline must also be able to gain an asset value at the end of the third year of at least 160.0375 million euros after the acquisition of the slot bundle. The maximum price  $p^*$  that can be paid under these conditions is calculated by the following linear optimization approach. In this context, the airline forecasts cash inflows by using the slot bundle with an amount of 4 million euros in  $t = 1$ , 3 million euros in  $t = 2$ , and 2 million euros in  $t = 3$ :

max. VIU; VIU: = p

80 T + F<sub>0</sub> - L<sub>0</sub> + p

≤ 30

-40 T - 1.05 F<sub>0</sub> + 1.1 L<sub>0</sub> + F<sub>1</sub> - L<sub>1</sub>

≤ 30 + 4

-40 T - 1.05 F<sub>1</sub> + 1.1 L<sub>1</sub> + F<sub>2</sub> - L<sub>2</sub>

≤ 30 + 3

G<sub>3</sub> - 40 T - 1.05 F<sub>2</sub> + 1.1 L<sub>2</sub>

≤ 30 + 2

T

≤ 1

G<sub>3</sub>

≥ 160.0375

G<sub>3</sub>, T, F<sub>0</sub>, F<sub>1</sub>, F<sub>2</sub>, L<sub>0</sub>, L<sub>1</sub>, L<sub>2</sub>, p

≥ 0

The maximum price that can be paid for the bundle of four slots is  $p^* = 7.8829$  million euros. The valuation program includes the tangible asset investment, bundle of four slots, loan in the first and second years, and financial asset investment in the third year. In Table 3, the described valuation program is shown with all cash streams that occur during the four years.

Table 3. Valuation program

Year	t = 0	t = 1	t = 2	t = 3
Tangible asset T	-80	40	40	40
Autonomous cash flow $b_t$ + slot cash flow $g_{Kt}$	30	34	33	32
Price $p^*$	-7.8829			
Loan $L_0$	57.8829	-63.6712		
Financial asset $F_1$		-10.3288	10.8453	
Financial asset $F_2$			-83.8453	88.0375
Asset value				160.0375

As shown in the example, the simplex algorithm not only determines the maximum price the airline can pay for the slot bundle, which is necessary in order to prevent a disadvantageous investment, but additionally provides the airline’s optimal investment and financing program, including cash flows from the slot bundle and the acquisition price  $p^*$ .

Consequently, in cases when the airline does not know the IRR of the best alternative investment, linear programming can deliver the relevant maximum price. Applying linear programming requires first determining the maximum success represented by the maximum asset value of the airlines’ combination of financing and investment options without considering the purchase of the slot bundle. Therefore, the airline has to identify current investment and finance options, and by solving a simplex algorithm, calculate the attainable value at the end of the investment period. Second, the airline must determine the maximum price payable in state  $t = 0$  by solving the second sim-

plex algorithm, taking into account the additional cash flow stream generated by the slot bundle.

Although linear programming was only shown for the purchase decision, it can likewise be applied to the decision to sell the slot bundle. For this purpose, the airline has to determine the optimum value  $AV^*$  of its baseline program as the first step, including the cash flow stream generated by the slot bundle.<sup>18</sup> Second, the valuation program must be slightly modified: since the airline now considers selling the slot bundle, the valuation model has to derive the minimum price  $V$  that allows the airline to retain the optimum  $AV^*$ , taking into account that the cash flow stream generated by the slot bundle is no longer considered. Consequently, the valuation program can be calculated by the following linear optimization program:<sup>19</sup>

$$\begin{aligned}
 &\min. V; V := p \\
 &-\sum_{j=1}^m g_{j0} \cdot x_j + G_0 - p \leq b_0 \\
 &-\sum_{j=1}^m g_{jt} \cdot x_j + G_t \leq b_t - g_{kt} \quad \forall t \in \{1, 2, \dots, n\} \\
 &-\sum_{t=0}^n w_t \cdot G_t \leq -AV^* \\
 &x_j \leq x_j^{\max} \quad \forall j \in \{1, 2, \dots, m\} \\
 &G_t, p, x_j \geq 0 \quad \forall t \text{ and } j
 \end{aligned}$$

### III. VALUATION IN PRACTICE

#### A. SUBJECTIVE DECISION FIELDS

As the core concern of this article, the determination of the VIU is based on the subjective investment and financing alternatives the individual has at the moment of valuation. Unlike this subjective perspective, discounted cash flow models<sup>20</sup> in financial theory are based on approaches such as the Capital Asset

<sup>18</sup> Applying the same numerical example to the disposal situation with the investment in asset  $T$ , the autonomous cash flow  $b_t$  + the slot cash flow  $g_{kt}$  reveals an optimum  $AV^* = 169.5975$  million euros.

<sup>19</sup> The continuation of the numerical example for the sale of the slots delivers the minimum price  $p^* = 7.8829$  million euros.

<sup>20</sup> With respect to discounted cash flow models, see generally COPELAND et al., *supra* note 6, at 62–67.

Pricing Model (CAPM),<sup>21</sup> which relies on certain premises that are necessary for determining objective market values in market equilibrium. As an example, Deutsche Lufthansa AG applies the cash value-added concept as its companywide value-based management measure.<sup>22</sup> In this model, a certain capital expenditure generates additional value if cash flows exceed the cost of capital applied. Cost of capital applied represents the return on investment expected by investors and is determined using the weighted average cost of capital (WACC) which is calculated based on the assumptions of the CAPM. Critical assumptions of the CAPM include, for example, identical information of market participants and, accordingly, homogeneous expectations. Consequently, all investors hold identical portfolios of risky assets (that is, the market portfolio). Additionally, derived market prices are based on the presumed possibility of borrowing and investing money at a constant interest rate that is available to any investor in the market.

In contrast, it is necessary to take into account the real alternative investment and financing decisions of the subject conducting the valuation (that is, the subjective portfolio), so as to derive values that are relevant to decisions. This is why, in short, the VIU can also be referred to as the decision value. Thus, in principle, there are no limitations with respect to the number of possible realizations since they are specific to the respective individual. Hence, for the purpose of valuing take-off and landing rights, the airline's alternatives—the subjective decision field—could be represented by, for example, the redemption of a credit, the investment in slots at another airport, and the lease of aircrafts to competitors, but also any other investment or financing project the airline is able to undertake. These alternatives have an effect on both the cash flows generated by the usage of the slots (for example, tangible asset investment *T* in the linear programming example shown above) and on the relevant discount rate that is applied in the present value technique.

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<sup>21</sup> For specifications regarding the CAPM, see John Lintner, *The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets*, 47 REV. OF ECON. & STATS. 13 (1965); Harry Markowitz, *Portfolio Selection*, 7 J. FIN. 77 (1952); Jan Mossin, *Equilibrium in a Capital Asset Market*, 34 ECONOMETRICA 768 (1966); William F. Sharpe, *Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*, 19 J. FIN. 425, 427, 433–34 (1964).

<sup>22</sup> See DEUTSCHE LUFTHANSA AG, *supra* note 10, at 39.

## B. BUNDLE PROBLEM

As mentioned in Section II. A., the VIU must generally be calculated not for a single slot, but for a bundle of at least four slots. This may constitute a problem because the valuation computes the price limit for the four slots together. Yet, it is frequently the case that four slots cannot be purchased in one transaction. In other words, the airline has the challenging situation of negotiating more than one slot transaction. For example, the airline may be engaged in two negotiations (first, buying a take-off and a landing right at Airport A, and second, buying a landing and a take-off right at Airport B), for which the sum of the two individual price negotiations must not exceed the VIU of the bundle of four slots. In many cases, it will not be possible for the airline to conduct parallel negotiations, such that successive acquisitions of the four slots are often the only practical solution. In these situations, the airline bears the risk of being able to buy part of the slot bundle in the first negotiation but not the entire bundle because the price limit for the bundle is exceeded in the second negotiation. However, in such a case, there are two ways out of the dilemma, either by buying other slots as a substitute (for example, landing and take-off rights at Airport C) or reselling those parts of the slot bundle that have already been purchased.

Apart from the situation in which at least four slots are valued altogether, constellations may also be found where a VIU must be calculated for less than four slots. This can occur, for example, if an airline in the European Union receives take-off and landing rights in Stockholm and landing and take-off rights in Frankfurt from the national authorities, but wishes to buy take-off and landing rights at London Heathrow for flying Heathrow-Frankfurt instead of Stockholm-Frankfurt. In order to determine the VIU of the two Heathrow slots that the airline plans to buy, only the cash flow changes that occur through the slot acquisition, in comparison to the initial constellation, must then be taken into account. Section III. D. 1. presents an exemplary specification of this practical valuation task.

## C. PROBLEM OF UNCERTAINTY

In addition to considering the subjective alternatives at a given point in time for the determination of decision-relevant values, the airline undertakes the difficult task of estimating the elements needed to calculate the slot value. These include, for

example, the turnover earned from passenger and freight transport, expenses associated with aircraft and airports, and the discount rate. All of these quantities will occur in the future, that is, they are uncertain and must therefore be forecasted. This uncertainty cannot be eliminated, but it is useful to integrate the likely effects visibly into the slot value. A risk analysis based on a Monte Carlo simulation<sup>23</sup> is a particularly suitable instrument for this purpose. It computes the statistical distribution of the VIU, based on the assumed distributions of the elements used for the valuation. In order to conduct the risk analysis, the airline must perform three steps. First, it must estimate the possible range of magnitudes of each element entering into the valuation (turnover, expenses, discount rate) and the probability of each magnitude occurring. Second, for each uncertain element, a particular amount is selected at random from the distributions assumed in step one, and the value of the slot bundle is calculated using these selected amounts. This selection of magnitudes is repeated several times with the aid of a computer, and the value calculated on the basis of each selection is saved. If this process is repeated often enough, the frequency distribution of the VIU evolves from these calculations. After this frequency distribution has been determined, the interpretation of the results follows as step three. In addition to the calculation of means and medians, it is particularly useful to transform the results into diagrams, for example into a histogram or a risk profile. The results of the risk analysis are of considerable significance to the airline because they show the complete range of possible outcomes of the VIU of the slot bundle and their respective probabilities. It is the airline's individual decision as to how to apply these results to identify a certain VIU, that is, to set a specific price limit for the slot bundle. For example, a risk-neutral airline will choose the mean VIU as the price limit, but an airline that prefers to take certain risks will choose a price limit that exceeds the mean. The application of a Monte Carlo simulation in practice will be shown in Section III.D.2.

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<sup>23</sup> See generally David B. Hertz, *Risk Analysis in Capital Investment*, 42 HARV. BUS. REV. 95 (1964); William Gibson & Peter Morrell, *Theory and Practice in Aircraft Financial Evaluation*, 10 J. AIR TRANSP. MGMT. 427 (2004).



## D. EXEMPLARY SLOT VALUE DETERMINATION

1. *The Consideration of Real Alternatives*

For the purpose of presenting some of the valuation situations that arise in practice, it is appropriate to refer again to the example introduced in Section II.A. as our base case: we consider an airline that owns a take-off and a landing right at Airport A and a landing and a take-off right at Airport B. The company forecasts that flying between A and B will lead to periodic net cash flows of three million euros that remain constant over the future periods one to three (for the sake of simplicity, we assume just three future periods of slot usage). As shown in Section II.A., the maximum price the airline should be willing to pay, considering the IRR of its best alternative investment of 4%, amounts to 8.33 million euros.

As a variation of the airline's realizable investment portfolio at a given moment, let us now assume that immediately (that is, one logical second) after the acquisition of the slot bundle for serving the route A to B (in the analysis below, it is assumed that the airline had to pay its maximum subjective acquisition price of 8.33 million euros at locations A and B, so that the transaction is neither advantageous nor disadvantageous, but is, nevertheless, conducted), the airline has the chance to acquire a take-off and landing right at Airport C. The airline now considers whether it is financially advantageous to include destination C in its route network.

In a situation like this, the airline can—in addition to maintaining the status quo (that is, serving the route A to B)—choose among the following two investment strategies. First, the airline could consider whether it is advantageous to include Airport C as an additional destination in its flight plan with the consequence that Airport C could be approached from either Airport A or B (*Alternative (a)*). Second, the airline could consider whether it could also be beneficial to acquire the slots at Airport C, while at the same time selling the already acquired take-off and landing rights at, for example, Airport B (*Alternative (b)*). Hence, slots at Airport C would represent a substitute for the rights at Airport B. The appraisal of the two investment alternatives are carried out separately below:

Regarding *Alternative (a)*—the assessment of the strategy to include Airport C as an additional location in the flight plan—the airline forecasts constant periodic net cash flows of 5 million euros over the periods one to three. That is, adding Airport C to

the route network leads to a rise in the cash flow per period  $\Delta CF_t$  of two million euros. Because the airline knows that the IRR of its best alternative investment is 4%, the VIU of the bundle of landing and take-off rights at Airport C is calculated as:

$$(7) \text{ VIU} = \sum_{t=1}^n \frac{\Delta CF_t}{(1+i)^t}.$$

Entering the data from the example, it follows:

$$\text{VIU} = \sum_{t=1}^3 \frac{2 \text{ million euros}}{(1.04)^t}$$

$$= 5.55 \text{ million euros.}$$

That is, for the two slots at Airport C, the airline can afford to pay a maximum price of 5.55 million euros. If it paid more, the acquisition of the slots at Airport C as an additional destination would be disadvantageous, and the transaction should not be executed.

In order to illustrate the decision process, it is furthermore assumed that the airline can acquire the take-off and landing right at Airport C for four million euros. Consequently, since the actual acquisition price is lower than the subjective maximum price of 5.55 million euros, the acquisition and additional operation to Airport C is beneficial to the airline (it is assumed that the airline has four million euros available in liquid funds which would be invested and yield 4% if the acquisition does not take place). Thus, after the airline has also acquired the slots at Airport C, its flight operations between A, B, and C generate periodic net cash flows of 5.0 million euros with a VIU of 13.88 million euros, while the total acquisition price amounts to 12.33 million euros. Hence, *Alternative (a)* yields an IRR of 10.50%. An overview of the investment characteristics with respect to *Alternative (a)* is shown in Table 4.

**Table 4. Investment overview *Alternative (a)***

Year		t = 0	t = 1	t = 2	t = 3
IRR of best alternative investment	4.00%				
Acquisition of slots at Airports A and B		-8.33			
Acquisition of slots at Airport C		-4.00			
Slot cash flows			5.00	5.00	5.00
Value in use (discount rate 4.00%)	13.88				
Total acquisition price	12.33				
IRR <i>Alternative (a)</i>	10.50%				

With respect to *Alternative (b)*—the valuation of the acquisition of the slots at Airport C while selling the slots at Airport B—it is assumed that the switch of Airports from B to C leads to a rise in the net cash flow per period of 0.5 million euros over the periods one to three. Additionally, because the airline can finance the acquisition of the slots at Destination C from its liquid funds, it can reinvest the money that is received from the disposal of the slots at Airport B in a financial asset yielding the original best rate of 4%. For quantification reasons it is then assumed that, besides the acquisition of the slots at Destination C amounting to four million euros, the slots at Airport B can be sold for 2.5 million euros. Consequently, the reinvestment of 2.5 million euros yields periodic interest payments of 0.1 million euros and the redemption of 2.5 million euros in  $t = 3$ .

Since the IRR of *Alternative (a)* of 10.50% exceeds the IRR of the best alternative investment of 4% at the starting point of the example, it is the new benchmark for assessing whether *Alternative (b)* is preferred to *Alternative (a)*. Applying the rate of 10.50% in order to determine the subjective VIU of *Alternative (b)* reveals a value of 10.73 million euros. Given that the acquisition of the slots at Airport C requires the investment of 4.0 million euros and the initial investment in the slots at Airports A and B of 8.33 million euros,<sup>24</sup> *Alternative (b)* is not beneficial compared to the realization of *Alternative (a)* since the invested amount exceeds the airline's VIU. Table 5 summarizes the results of the appraisal of *Alternative (b)*.

**Table 5. Investment overview *Alternative (b)***

Year		t = 0	t = 1	t = 2	t = 3
IRR of best alternative investment	10.50%				
Acquisition of slots at Airports A and B		-8.33			
Acquisition of slots at Airport C		-4.0			
Disposal of slots at Airport B		2.5			
Slot cash flows			3.5	3.5	3.5
Cash flows from reinvestment		-2.5	0.1	0.1	2.6
Value in use (discount rate 10.50%)	10.73				
Total amount invested	12.33				
IRR <i>Alternative (b)</i>	3.54%				

The example presented above underlines the critical importance of considering the valuation subject's individual invest-

<sup>24</sup> Because the proceeds from the disposal of the slots at Airport B are reinvested, the total amount invested remains at a constant level of 12.33 million euros.

ment and financing options at a given point in time. If it is the intention to determine subjective maximum (purchase decision) or minimum (disposal decision) prices that prevent disadvantageous situations, any change in the range of an individual's possible investment and financing realizations affecting both cash-flow streams and the discount rate must be taken into account.

## 2. The Consideration of Uncertainty

In the following analysis, the valuation of slots using risk analysis is shown for the example already presented in Section III.D.1., which has been modified slightly for this purpose:

We still assume that an airline knows the IRR of its best alternative investment, which is 4%. It is further assumed that this rate is constant over time and is also certain. With respect to the future cash flows that can be earned from the slot bundle, the airline forecasts, based on market research and existing freight transport contracts, a certain surplus of three million euros in  $t = 1$ . Due to the difficulty of predicting future market developments, the earnings in the following periods are uncertain, so that the airline forecasts earnings for the periods  $t = 2$ ,  $t = 3$ , and  $t = 4$  to  $\infty$ , are subject to a normal distribution with the expected values (3.25, 3.50, 3.75) (in million euros) and the standard deviations (0.25, 0.50, 0.75). From ten thousand calculation processes, the following histogram of the slot bundle's VIU is obtained.

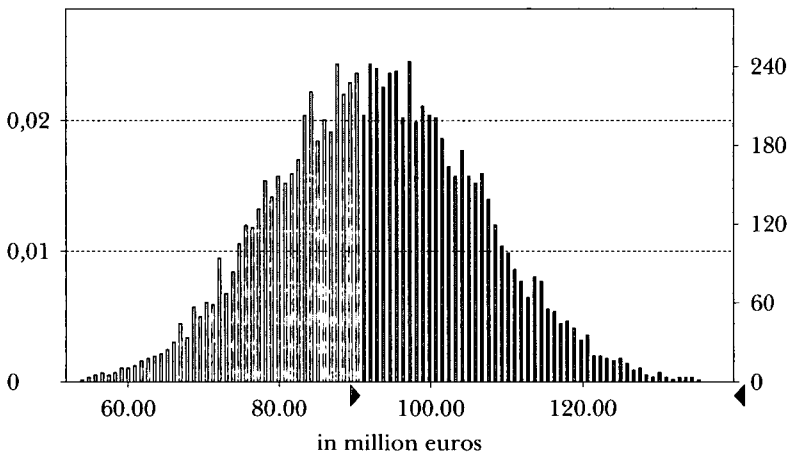


Table 6: Histogram of VIU

The mean VIU amounts to 92.3 million euros. A presentation of the risk-analysis results equivalent to the histogram yields the

risk profile shown in the following table. It demonstrates the likelihood of the VIU exceeding a certain level on the abscissa. For example, the likelihood is 56.06% that the value of the slots will exceed ninety million euros.

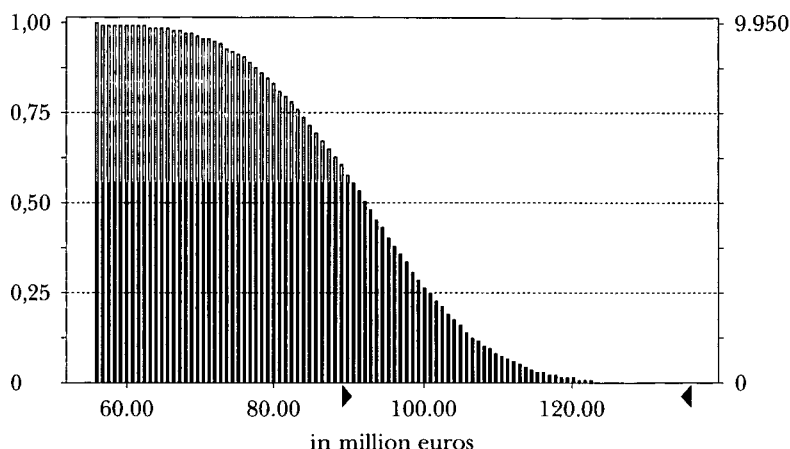


Table 7: Risk profile of VIU

As explained above, it depends on the airline's individual risk preference whether it chooses a maximum price in the acquisition process below the mean value of 92.3 million euros (risk averse), a maximum price that is equal to the mean value (risk neutral), or an amount exceeding the mean value (risk friendly).

#### IV. SUMMARY

Airlines that intend to buy take-off or landing rights, as is possible in the United States and United Kingdom, must, first of all, determine a value for these slots. The aim is not to calculate an objective market value, but to identify the subjective VIU that the slots have for the airline in question. The VIU indicates the maximum price the airline can spend on the slots without the transaction being disadvantageous. In order to compute this price limit, a simple present value technique can be used if the airline knows both its best alternative investment compared to the slot acquisition and the IRR of this investment. The latter is used as the discount rate in the present value technique. If the airline does *not* know the best alternative investment because of its very complex investment and financing portfolio, the slot valuation can be based on linear programming, that is, the ZGPM. Two problems occur within the context of the valuation. First, the bundle problem results from the fact that the value can gen-

erally be calculated only for a bundle of at least four slots, but purchasing these slots is often possible only in several individual transactions. The sum of the prices negotiated in these individual transactions must not exceed the VIU, which takes into account the airline's alternative investment and financing projects. Furthermore, there is the problem of uncertainty, because the valuation must always be based on magnitudes that are obtained in the future. This uncertainty cannot be eliminated, but its effects on the slot value can be identified with the help of a Monte Carlo simulation.

